MATH2403B____ 2-10-2000 TEST 2 NAME:_____
INSTRUCTOR:_____ STUDENT NO:_____

Show all work.

1. A vertical spring with a spring constant equal to 108 lb/ft has a 96 lb weight attached to it. A dashpot (or a shock absorber) with a damping coefficient c = 36 lb-sec/ft is attached to the weight. Suppose that a downward force of f(t) =72 $cos(6t)$ is applied to the weight. If the weight is released from rest at the equilibrium position at time $t = 0$,
(A) show that the differential equation governing the displacement $x(t)$ is

$$x''(t) + 12\,x(t) + 36\,x(t) = 24\,cos(6t)$$

where g = 32 ft/sec^2 is used.
(B) Find the solution satisfying the equation established in Part (A) and the given initial conditions.

2. An inductor of 5 henries is connected in series with a capacitor of 1/180 farads, a resistor of 60 ohms and a voltage-supply given by E(t) = 120 $cos(6t)$ in volts. Suppose that both the charge Q and the current I are zero initially.
(A) Show that the differential equation governing the charge Q(t) is

$$Q''(t) + 12Q' + 36Q(t) = 24\,cos(6t).$$

(B) Find the charge Q(t) satisfying the equation of Part (A) and the given initial conditions.

3. Consider the system

$$x'(t) = y(t), \qquad y'(t) = -36\,x(t) - 12\,y(t).$$

(A) Write the system in the form $\vec{x}'(t) = A\vec{x}(t)$.
(B) Find the eigenvalues of A.
(C) Guess a form for the general solutios $x_g(t),\ y_g(t)$.
(D) Hence find $x_g(t),\ y_g(t)$.

4. Solve $x'(t) = y(t), \quad y'(t) = -36\,x(t) - 12y(t) + 24\,cos(6t)$
with the initial conditions $x(0) = 0,\ y(0) = 0$.
(B)Write your solutions in vector form.
(C)Write your solutions in matrix form. What is the fundamental matrix you have found?