Name________________________________ 1 August 2000
Teaching Assistant____________________ Final Exam
Math 2403AC Andrew/Green

Instructions: 1. Closed book, calculators may be used.
2. You may use one 8.5 by 11 inch formula sheet.
3. Show your work and explain your answers and reasoning.

1. (20) a. Calculate the general solution of $x'' + 4x' + 29x = 0$.

b. Use the method of undetermined coefficients to find a particular solution of $x'' + 4x' + 29x = 136\cos(3t)$.

c. Calculate the solution of the initial value problem

$$\begin{aligned} x'' + 4x' + 29x &= 136\cos(3t) \\ x(0) &= 8 \\ x'(0) &= 0 \end{aligned}$$

2. (20) Use the eigenvalue method to solve

$$\begin{aligned} x' &= x + 2y \\ y' &= -4x - 5y \\ x(0) &= 1 \\ y(0) &= 2 \end{aligned}$$

3. (20) a. Use Euler's method with step size $h = .1$ to approximate $y(.3)$, given that $y' = 2xy$, $y(0) = 1$.

b. Find the general solution of $\dfrac{dy}{dx} = (64xy)^{1/3}$.

4. (20) a. Find all critical points of the system

$$\begin{aligned} \frac{dx}{dt} &= xy - 2 \\ \frac{dy}{dt} &= x - 2y \end{aligned}$$

b. Determine the type and stability of each critical point of the system from part a.

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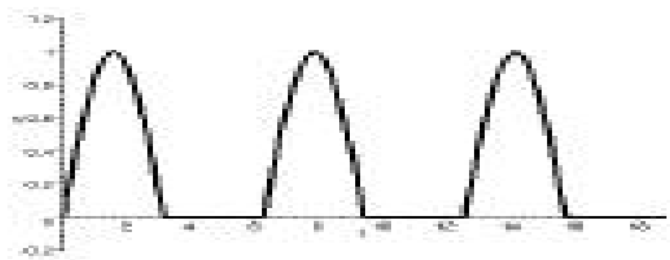
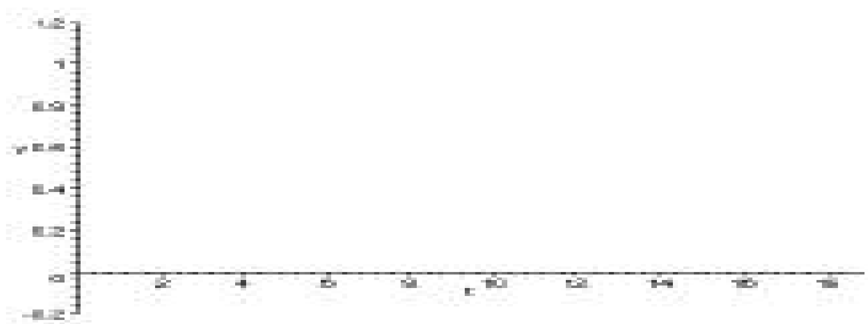
5. (20) a. Sketched below is the graph of the "half-wave rectification" of the sine function

$$h(t) = \begin{cases} \sin(t), & \sin(t) \geq 0 \\ 0, & \sin(t) < 0 \end{cases}$$



Compute the Laplace transform of *h(t)*.

b. On the axes provided, sketch the graph of $f(t) = h(t) + u(t-\pi)h(t-\pi)$.



c. Compute the Laplace transform of *f(t)*.

6. (20) In this problem we seek a series solution $y = \sum_{n=0}^{\infty} a_n x^n$ to the differential equation $y'' - xy = 0$.

a. Find a recurrence relation satisfied by the coefficients a_n.

b. Now, assuming $y(0) = 1$, $y'(0) = 0$, use your recurrence relation to write down the terms in the series solution $y = \sum_{n=0}^{\infty} a_n x^n$, up to and including the term $a_{12}x^{12}$.

c. Still assuming $y(0) = 1$, $y'(0) = 0$, write the series solution $y = \sum_{n=0}^{\infty} a_n x^n$. (I.e., you must determine the general form of the coefficient a_n.)

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Answers.

1. a. $x_g(t) = Ae^{-2t}\cos(5t) + B\,e^{-2t}\sin(5t)$ b. $x_p(t) = 5\cos(3t) + 3\sin(3t)$

   c. $x(t) = 3e^{-2t}\cos(5t) - \dfrac{3}{5}e^{-2t}\sin(5t) + 5\cos(3t) + 3\sin(3t)$

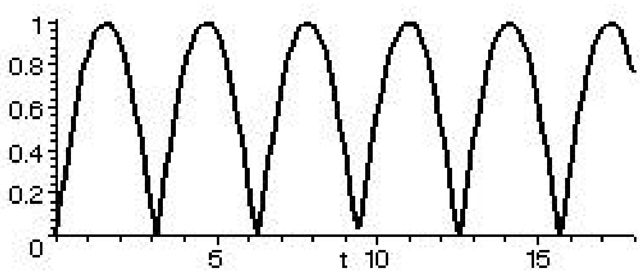
2. $x(t) = -3\begin{pmatrix}1\\-2\end{pmatrix}e^{-3t} + 4\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}$

3. a. 1.0608 b. $y=\left(2x^{1/3} + D\right)^{3/2}$

4. (2,1) is an unstable saddle point, (-2,-1) is an asymptotically stable spiral point.

5. a. $H(s) = \dfrac{1}{1-e^{-\pi s}}\dfrac{1}{s^2+1}$ c. $F(s) = \dfrac{1+e^{-\pi s}}{1-e^{-\pi s}}\dfrac{1}{s^2+1}$

   b.



6. a. $a_{k+2} = \dfrac{a_{k-1}}{(k+2)(k+1)}, \quad k \geq 1$

   b.
$$1 + \frac{1}{(3)(2)}x^3 + \frac{1}{(3\cdot 6)(2\cdot 5)}x^6 + \frac{1}{(3\cdot 6\cdot 9)(2\cdot 5\cdot 8)}x^9 + \frac{1}{(3\cdot 6\cdot 9\cdot 12)(2\cdot 5\cdot 8\cdot 11)}x^{12}$$

   c. $1 + \displaystyle\sum_{n=1}^{\infty}\frac{1}{(3\cdot 6\cdots 3n)(2\cdot 5\cdots(3n-1))}x^{3n}$