Mathematics 2403 Hour Examination
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Directions: Do all problems. Show your work and justify your answers. Calculators are allowed, but this is a closed book examination.

1 (36) Find all solutions to each of the following equations:

a. $y'' - 3y' - 10y = 0$

b. $y'' - 3y' - 10y = 20e^{5x}$

c. $y'' - 3y' - 10y = \cos(5x)$

2. (12) Solve the initial value problem $y'' - y = \sin x, y(0) = 0, y'(0) = 1.$

3. (12) Show that the set $\{e^{2x}, e^{3x}\}$ is linearly independent.

4. (12) If $(D^2 + 4)^2 D^3 (D - 2)^4 y = 0$, what is the form of the general solution to this equation? (You needn't determine the constants; remember that D means "differentiate".)

5. (28) An unforced mass-spring system without damping has equation $4x'' + 9x = 0.$

a. (8) Find the circular frequency and the period of this system.

b. (10) Find the general (real) solution to the equation $4x'' + 9x = 0.$

c. (10) If this system is set in motion with an initial displacement of $\frac{1}{2}$ and initial velocity of $\frac{1}{2}$, what is the amplitude of the motion?

Answers. 1. a. $y = C_1 e^{5x} + C_2 e^{-2x}$ b. $y = C_1 e^{5x} + C_2 e^{-2x} + \frac{20}{7} x e^{5x}$

$$y = C_1 e^{5x} + C_2 e^{-2x} - \frac{7}{290}\cos(5x) - \frac{3}{290}\sin(5x)$$

2. $y = \frac{3}{4}e^{x} - \frac{3}{4}e^{-x} - \frac{1}{2}\sin(x)$

4.
$$C_1 \sin(2x) + C_2 \cos(2x) + C_3 x\sin(2x) + C_4 x\cos(2x) + C_5 + C_6 x + (C_8 + C_9 x)e^{2x}$$

5. a. circular frequency = 3/2, period = $4\pi/3$. b. $C_1 \cos\left(\frac{3t}{2}\right) + C_2 \sin\left(\frac{3t}{2}\right)$

c. $\sqrt{13}/6$