

## Mathematics 2403 Hour Examination

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**Directions:** Do all problems. Show your work and justify your answers. Calculators are allowed, but this is a closed book examination.

1 (36) Find all solutions to each of the following equations:

a.  $y'' - 3y' - 4y = 0$

b.  $y'' - 3y' - 4y = e^{4x}$

c.  $y'' - 3y' - 4y = \sin(4x)$

2. (12) Solve the initial value problem  $y'' + y = e^x$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .

3. (12) Suppose that  $y_1'' + 2y_1' + 2y_1 = e^x$  and that  $y_2'' + 2y_2' + 2y_2 = e^{-x}$ . Give a solution to  $y'' + 2y' + 2y = \frac{e^x + e^{-x}}{2}$ .

4. (12) If  $(D + 7)^2(D^2 + 25)D^4 y = 0$ , what is the form of the general solution to this equation? (You needn't determine the constants; remember that  $D$  means "differentiate".)

5. (28) An unforced mass-spring system without damping has equation  $36x'' + 25x = 0$ .

a. (8) Find the circular frequency and the period of this system.

b. (10) Find the general (real) solution to the equation  $36x'' + 25x = 0$ .

c. (10) If this system is now damped, so that the equation for the system becomes  $36x'' + cx' + 25x = 0$  with  $c > 0$ , what is the value of  $c$  which gives us critical damping of the motion?

### Answers.

1. a.  $y = C_1 e^{4x} + C_2 e^{-x}$  b.  $y = C_1 e^{4x} + C_2 e^{-x} + \frac{1}{5} x e^{4x}$

c.  $y = C_1 e^{4x} + C_2 e^{-x} + \frac{3}{136} \cos(4x) - \frac{5}{136} \sin(4x)$

2.  $y = \frac{1}{2} e^x + \frac{1}{2} \sin(x) - \frac{1}{2} \cos(x)$  3.  $\frac{1}{2}(y_1 + y_2)$

4.  $(C_1 + C_2 x) e^{-7x} + C_3 \sin(5x) + C_4 \cos(5x) + C_5 + C_6 x + C_7 x^2 + C_8 x^3$

5. a. circular frequency =  $5/6$ , period =  $12/\sqrt{5}$ . b.  $A \cos\left(\frac{5t}{6}\right) + B \sin\left(\frac{5t}{6}\right)$ . c.  $c = 60$ .