Mathematics 2403 Hour Examination
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Directions: Do all problems. Show your work and justify your answers. Calculators are allowed, but this is a closed book examination.

1. (36) Consider the homogeneous system $\begin{aligned} x' &= 2x + y \\ y' &= 6x + y \end{aligned}$

a. (9) Write this system in matrix form $\mathbf{x}' = A\mathbf{x}$ and find the eigenvalues for A.

b. (9) Find the general solution for this system.

c. (18) Find the general solution for the nonhomogeneous system $\begin{aligned} x' &= 2x + y + e^{2t} \\ y' &= 6x + y \end{aligned}$

2. (24) For each of the following systems, specify the **form** for the solution for the system. Do **not** compute the coefficients. (For example, the **form** for the general solution of $\begin{aligned} x' &= x \\ y' &= -5y \end{aligned}$ is $\begin{aligned} x &= A_1e^t + A_2e^{-5t} \\ y &= B_1e^t + B_2e^{-5t} \end{aligned}$ In this example A_2 and B_1 are zero, but I don't want you to have to compute any of the A's or the B's.) [Hint: Look again at problem 1.]

a. (8) $\begin{aligned} x' &= x + y \\ y' &= -5x - y \end{aligned}$ $\mathbf{x} = ?$ (General solution)

b. (8) $\begin{aligned} x' &= 2x + y \\ y' &= 6x + y + e^{-t} \end{aligned}$ $\mathbf{x_p} = ?$ (A particular solution)

c. (8) $\begin{aligned} x' &= 2x + y \\ y' &= 6x + y + \sin 3t \end{aligned}$ $\mathbf{x_p} = ?$ (A particular solution)

3. (16) The system $\begin{aligned} x' &= x + 2 \\ y' &= x - y + 2 \end{aligned}$ has fundamental matrix $U = \begin{bmatrix} 2e^t & 0 \\ e^t & e^{-t} \end{bmatrix}$. Find a particular solution for this system.

5. a. (8) Find an explicit formula for e^{tA}, where $A = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$.

b. (8) Find an explicit formula for e^{tA}, where $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$.

c. (8) Solve the initial value problem $\begin{bmatrix} x' = 2y \\ y' = 0 \end{bmatrix}$ $x(0) = 5, y(0) = -3.$

Answers.

1. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, $\lambda = 4,1$ b. $\begin{pmatrix} x \\ y \end{pmatrix} = C_1\begin{pmatrix} 1 \\ 2 \end{pmatrix}e^{4t} + C_2\begin{pmatrix} 1 \\ -3 \end{pmatrix}e^{-t}$

c. $\begin{pmatrix} x \\ y \end{pmatrix} = C_1\begin{pmatrix} 1 \\ 2 \end{pmatrix}e^{4t} + C_2\begin{pmatrix} 1 \\ -3 \end{pmatrix}e^{-t} + \begin{pmatrix} -\frac{1}{6} \\ -1 \end{pmatrix}e^{2t}$

2. a. $x = A\cos(2t) + B\sin(2t)$, $y = C\cos(2t) + D\sin(2t)$.

b. $\begin{pmatrix} A \\ B \end{pmatrix}e^{-t} + \begin{pmatrix} C \\ D \end{pmatrix}te^{-t}$ c. $x = A\cos(3t) + B\sin(3t)$, $y = C\cos(3t) + D\sin(3t)$

3. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2e^{t} - 2 \\ e^{t} - e^{-t} \end{pmatrix}$ 5. a. $\begin{pmatrix} e^{2t} & 0 \\ 0 & e^{5t} \end{pmatrix}$ b. $\begin{pmatrix} 1 & 2t \\ 0 & 1 \end{pmatrix}$ c. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2t \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ -3 \end{pmatrix}$