Name______________________________Recitation Leader_________________________

Mathematics 2403 Hour Examination
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Directions: Do all problems. Show your work and justify your answers. Calculators are allowed, but this is a closed book examination.

1. (20) Find the general solution to $y' + (\tan x)y = \sin x$. [Hint: $\int \tan x dx = \ln(\sec x) + C$.]

2. (20) Find the general solution to $\frac{dy}{dx} = \cos(2x) + y\cos(2x)$.

3. (60) Consider the system $\begin{aligned} x' &= -x - y \\ y' &= 4x + 3y + e^t \end{aligned}$

a. (20) Write this system in matrix-vector form $\mathbf{x}'(t) = A\mathbf{x}(t) + \mathbf{b}(t)$ and find the eigenvalues for the matrix A.

b. (20) Find the matrix exponential e^{tA}, where $A = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$.

c. (20) Find a particular solution to this system.

Answers.

1. $y = \cos(x)\ln(\sec(x)) + C\cos(x)$

2. $y = Ke^{\frac{\sin(2x)}{2}} - 1$

3. a. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ e^t \end{pmatrix}$, $\lambda = 1,1$

b. $e^{t\mathbf{A}} = e^t e^{t(\mathbf{A}-\mathbf{I})} = e^t\left(\mathbf{I} + t(\mathbf{A}-\mathbf{I})\right) = e^t \begin{pmatrix} 1-2t & -t \\ 4t & 1+2t \end{pmatrix}$

c. $\mathbf{x}_p = e^t \begin{pmatrix} -\frac{t^2}{2} \\ t^2 + t \end{pmatrix}$