Name________________________________ 20 July 2000
Teaching Assistant_____________________ "Hour" Test IV
Math 2403A Andrew

Instructions: 1. Closed book, calculators may be used.
2. You may use one 8.5 by 11 inch formula sheet.
3. Show your work and explain your answers and reasoning.

1. (30) On this page you will find three first order linear systems, each of which has a critical point at $(x_*, y_*) = (0,0)$. Write the system in the form $\begin{pmatrix} x' \\ y' \end{pmatrix} = \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}$, where **A** is a 2 by 2 matrix. Calculate the eigenvalues of **A**, and use them to match each system with its phase portrait, selected from the pictures on the next page. Please place your answers in the spaces provided.

a. $\begin{aligned} x' &= 4x + y \\ y' &= x + 2y \end{aligned}$ Eigenvalues______________ Phase Portrait______

b. $\begin{aligned} x' &= -x + 4y \\ y' &= -x - y \end{aligned}$ Eigenvalues______________ Phase Portrait______

c. $\begin{aligned} x' &= -2y \\ y' &= 8x \end{aligned}$ Eigenvalues______________ Phase Portrait______

2. (30) a. Locate all three of the critical points of the system

$$(**) \qquad \begin{aligned} x' &= 4y - x^2 y \\ y' &= x^2 - y \end{aligned}$$

b. Determine the matrix of the linearization at the critical point (x_*, y_*) for which both $x_* > 0$ and $y_* > 0$.

c. What type of critical point does the system (**) have at this (x_*, y_*)?

3. (40) Use the Laplace transform to solve the initial-value problem

$$x'' + 6x' + 25x = 34e^{-2t}$$
$$x(0) = 0, x'(0) = 2$$

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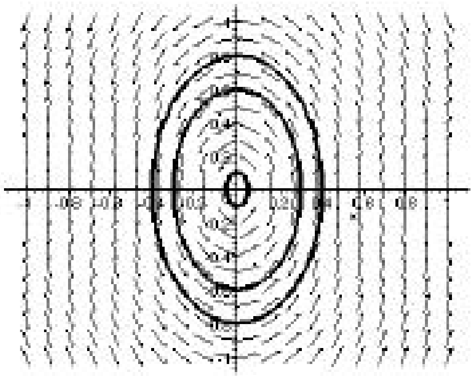
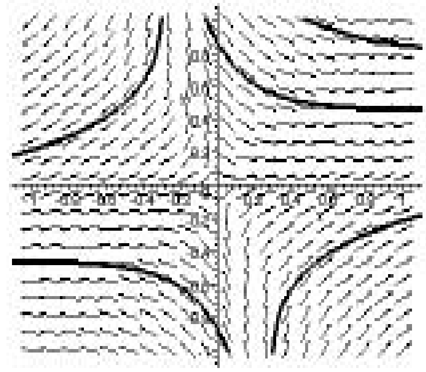
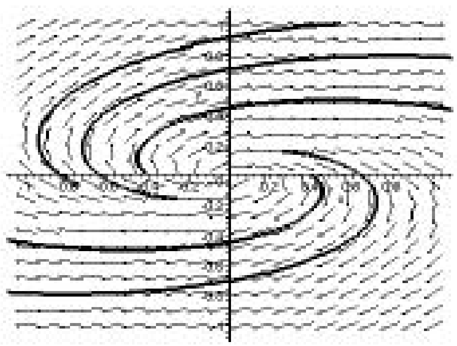
A.



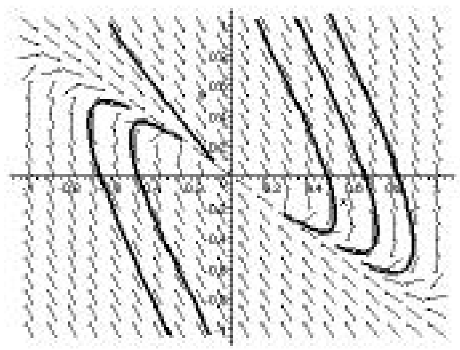
B.



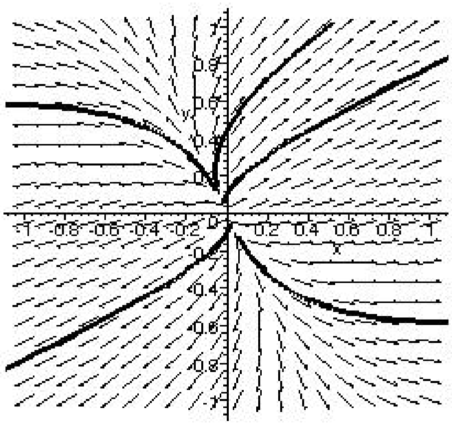
C.



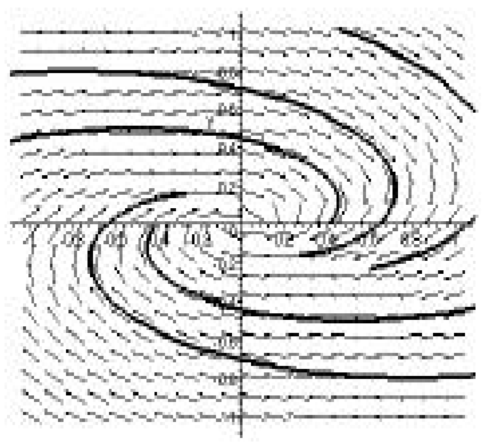
D.



E.



F.



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Answers.

1. a. $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$, $\lambda = 3 \pm \sqrt{2}$, Phase portrait E

   b. $\mathbf{A} = \begin{pmatrix} -1 & 4 \\ -1 & -1 \end{pmatrix}$, $\lambda = -1 \pm 2i$, Phase portrait F

   c. $\mathbf{A} = \begin{pmatrix} 0 & -2 \\ 8 & 0 \end{pmatrix}$, $\lambda = \pm 4i$, Phase portrait A

2. a. (0,0), (2,4), (- 2, 4)

   b. $\begin{pmatrix} -16 & 0 \\ 4 & -1 \end{pmatrix}$ c. Asymptotically stable node.

3.

$$\begin{aligned} X(s) &= \frac{38 + 2s}{(s+2)(s^2+6s+25)} \\ &= \frac{2}{s+2} + \frac{-2s - 6}{s^2+6s+25} \\ &= \frac{2}{s+2} - 2\frac{s+3}{(s+3)^2+16}, \quad \text{so} \\ x(t) &= 2e^{-2t} - 2e^{-3t}\cos(4t) \end{aligned}$$