

Math 4317
Hour Test 1
21 October 1994

1. a. Let A and B be non-empty sets in \mathbf{R} . Define $S = \{a+b: a \in A, b \in B\}$. Show that

$$\sup S = \sup A + \sup B$$

- b. Show that $\sqrt[3]{2}$ is an irrational number.
2. a. State the definition of “ K is a compact set in \mathbf{R}^p .”
- b. Use the definition of compact set to show that

$$K = \left\{ \frac{1}{n} : n \in \mathbf{N} \right\} \cup \{0\}$$

is compact in \mathbf{R} .

- c. Determine the set of cluster points and the boundary set of K .
3. a. Let A be a nonempty set in \mathbf{R}^p . Show that if $F \subseteq A$ and F is closed, then $F \subseteq (A \cap A')$, where A' denotes the set of cluster points of A .
- b. Show that the set of rational numbers \mathbf{Q} is neither open nor closed in \mathbf{R} .
4. True or false. Give a reason if you think it is true, and give a counterexample if you think it is false.
- Every countable infinite set has empty interior.
 - Every countable set in \mathbf{R} is closed in \mathbf{R} .
 - The union of two connected sets is connected.
 - The boundary of every closed connected set in \mathbf{R}^2 is connected.
 - Every uncountable set in \mathbf{R} must have at least one cluster point.