Math 4317
Hour Test 2
18 November 1994

Instructions:

1. Please begin each problem on a new sheet of paper.
2. Please do all four problems. Be sure to explain your work.
3. This is a closed book exam.

1. a. Give an $\varepsilon-\delta$ proof that $f(x) = \frac{1}{x}$ is continuous at a for every $a > 0$.

   b. Is f uniformly continuous on $(0,\infty)$?

   c. Is f uniformly continuous on $[1,\infty)$?

2. We say that a function f **satisfies a Lipschitz condition** on a set D if there exists a constant $K > 0$ such that $|f(x) - f(y)| < K\|x-y\|$ for all x, y in D.

   a. Prove that if f satisfies a Lipschitz condition on D, then f is uniformly continuous on D.

   b. Give an example of a function (and a domain D, of course) which is uniformly continuous on D but does not satisfy a Lipschitz condition on D.

3. Convince me that each of these sequences is convergent, and calculate the limit of any three of them.

   a. $x_n = \frac{n^3}{3^n}$

   b. $y_n = \left(1+\frac{1}{2^n}\right)^{(2^n)}$

   c. $z_n = \left(1+\frac{1}{2^n}\right)^{(n)}$

   d. $x_0 = 0, \quad x_1 = 1,$
   $x_{n+2} = \frac{1}{5}x_n + \frac{4}{5}x_{n+1}$

4. These are short-answer problems. Tell whether the statement is true or false and give a short explanation or counterexample.

   a. If (x_n) and (y_n) are divergent sequences of real numbers, then $(x_n y_n)$ is divergent also.

   b. If f is a bounded continuous function on a bounded domain D, then f is uniformly continuous on D.

   c. If f is a continuous function whose domain is **R** and *f(x)*   **Z** for every *x*, then f is constant.

   d. Every Cauchy sequence of rational numbers has a rational limit.