Math 4317
Sample Test 3
Problems selected from Tests in Math 4312, Fall 1988

1. By means of partial fractions, find a formula for the partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$. Does this series converge, and if so, to what?

2. Discuss the convergence of

   a. $\sum_{n=2}^{\infty} \frac{1}{n\log(n^2)}$

   b. $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$

   c. $\sum_{n=1}^{\infty} n^2 3^{-n}$

   d. $\sum_{n=1}^{\infty} \frac{1}{\left[n^2(2n+1)\right]^{1/2}}$

3. Let $b_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n-1} - \log(n)$.

   a. Show that (b_n) is a bounded increasing sequence, and hence has a limit γ.

   b. Letting $s_N = \sum_{n=1}^{N} \frac{1}{n}$ denote the partial sum of the harmonic series, calculate $\lim_{N\to\infty}(s_{2N} - s_N)$.

4. Calculate the radii of convergence of

   a. $\sum_{n=0}^{\infty} (n+2)3^n x^n$

   b. $\sum_{n=0}^{\infty} \left(\sqrt{n+1} - \sqrt{n}\right)x^n$

   c. $\sum_{n=0}^{\infty} 2^n x^{(n^2)}$